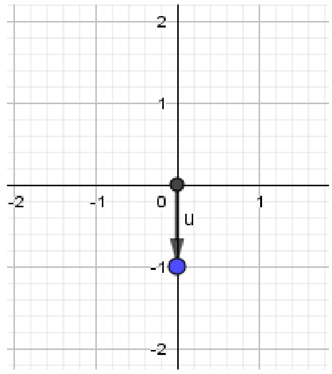
Úlohy k maturitní otázce č. 7 - příklad 7:

1. **Řešte rovnici**:



$z^3 = i^3$ binomická rovnice

$z^3 = -i$

$|-i| = 1$

$$z_k = \sqrt[3]{1}\left(cos\frac{270° + k.360°}{3} + i \cdot sin\frac{270° + k.360°}{3}\right)$$

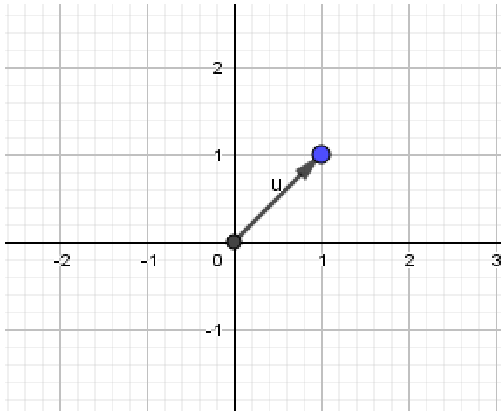
$$z_0 = \left(cos\frac{270°}{3} + i \cdot sin\frac{270°}{3}\right) = \cos 90° + i\sin 90° = 0 + i = i$$

$$z_1 = \left(cos\frac{270° + 1.360°}{3} + i \cdot sin\frac{270° + 1.360°}{3}\right) = \cos 210° + isin\,210° = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

$$z_2 = \left(cos\frac{270° + 2.360°}{3} + i \cdot sin\frac{270° + 2.360°}{3}\right) = \cos 330° + isin\,330° = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

Zkouška:

2. Řešte rovnici:



$z^4 - (1+i) = 0$

binomická rovnice

$z^4 = (1+i)$

$|1+i| = \sqrt{2}$ goniometrický tvar a = $\sqrt{2}\,(\cos 45° + i\sin 45°)$

$$z_k = \sqrt[4]{\sqrt{2}}\left(cos\frac{45° + k.360°}{4} + i \cdot sin\frac{45° + k.360°}{4}\right)$$

$$z_0 = \sqrt[8]{2}\left(cos\frac{45°}{4} + i \cdot sin\frac{45°}{4}\right)$$

$$z_1 = \sqrt[8]{2}\left(cos\frac{45°+1.360°}{4} + i \cdot sin\frac{45°+1.360°}{4}\right) = \sqrt[8]{2}\left(cos\frac{405°}{4} + i \cdot sin\frac{405°}{4}\right)$$

$$z_2 = \sqrt[8]{2}\left(cos\frac{45°+2.360°}{4} + i \cdot sin\frac{45°+2.360°}{4}\right) = \sqrt[8]{2}\left(cos\frac{765°}{4} + i \cdot sin\frac{765°}{4}\right)$$

$$z_3 = \sqrt[8]{2}\left(cos\frac{45°+3.360°}{4} + i \cdot sin\frac{45°+3.360°}{4}\right) = \sqrt[8]{2}\left(cos\frac{1125°}{4} + i \cdot sin\frac{1125°}{4}\right)$$

Zkouška např. pro

$$z_2 = \sqrt[8]{2}\left(cos\frac{45°+2.360°}{4} + i \cdot sin\frac{45°+2.360°}{4}\right) = \sqrt[8]{2}\left(cos\frac{765°}{4} + i \cdot sin\frac{765°}{4}\right)$$

$$z_2^4 = (\sqrt[8]{2}\left(cos\frac{765°}{4} + i \cdot sin\frac{765°}{4}\right)^4 = \sqrt{2}\left(cos4.\frac{765°}{4} + i \cdot sin4.\frac{765°}{4}\right) = \sqrt{2}(cos45° + isin\,45°) =$$
$$\sqrt{2}\left(\frac{\sqrt{2}}{2} + i.\frac{\sqrt{2}}{2}\right) = 1+i$$