

Řešte soustavy rovnic

$$1) \begin{aligned} x - y &= 27 & y &= x - 27 \\ x^2 - y &= 0 \\ \hline x^2 - x + 27 &= 0 & D = 1 - 4 \cdot 27 &= -107 \\ x_1, x_2 &= \emptyset \end{aligned}$$

$$2) \begin{aligned} 2x - y &= 0 & y &= 2x \\ y - x^2 &= 1 \\ \hline 2x - x^2 &= 1 \\ x^2 - 2x + 1 &= 0 \\ (x-1)^2 &= 0 \\ x &= 1 & y &= 2 \\ [xy] &= [1, 2] \end{aligned}$$

$$3) \begin{aligned} x^2 - y &= -3 \\ x - 2y &= 1 & x &= 1 + 2y \\ \hline 1 + 4y + 4y^2 - y &= -3 \\ 4y^2 + 3y + 4 &= 0 \\ D = 9 - 4 \cdot 4 \cdot 4 &= 0 \end{aligned}$$

$$4) \begin{aligned} x - 2y + xy &= 27 \\ x - y - 3 &= 0 & x &= y + 3 \\ \hline y + 3 - 2y + y(y+3) &= 27 \\ 3 - y + y^2 + 3y - 27 &= 0 \\ y^2 + 2y - 24 &= 0 \\ (y+6)(y-4) &= 0 \end{aligned}$$

$$\begin{array}{ll} y_1 = 6 & y_2 = 4 \\ x_1 = -3 & x_2 = 7 \\ [xy] = \{[-3, -6], [7, 4]\} \end{array}$$

$$5) \quad x^2 + 4y^2 = 10$$

$$\underline{x + 6y - 10 = 0} \quad x = 10 - 6y$$

$$100 - 120y + 36y^2 + 4y^2 = 10$$

$$40y^2 - 120y + 90 = 0$$

$$4y^2 - 12y + 9 = 0$$

$$\mathbb{D} = 144 - 4 \cdot 4 \cdot 9 = 0$$

$$y_{1,2} = \frac{12 \pm 0}{8} = \frac{3}{2}$$

$$x = 10 - 6 \cdot \frac{3}{2} = 10 - 9 = 1$$

$$[x, y] = [1, \frac{3}{2}]$$

$$6) \quad 2x^2 - 3y^2 - 5x - 2y = 26$$

$$\underline{x - y = 4} \quad x = y + 4$$

$$2(y^2 + 8y + 16) - 3y^2 - 5y - 20 - 6y = 26$$

$$2y^2 + 16y + 32 - 3y^2 - 5y - 20 - 6y - 26 = 0$$

$$-y^2 + 9y - 14 = 0$$

$$y_1 = 2 \quad y_2 = 7$$

$$y^2 - 9y + 14 = 0$$

$$x_1 = 6 \quad x_2 = 11$$

$$(y - 2)(y - 7) = 0$$

$$[x, y] = \{[6, 2], [11, 7]\}$$

$$7) \quad x^2 + y^2 + 4x - 2y = 0$$

$$\underline{x - y = 2} \quad x = 2 + y$$

$$4 + 4y + y^2 + y^2 + 8 + 4y - 2y = 0$$

$$2y^2 + 6y + 12 = 0$$

$$y^2 + 3y + 6 = 0$$

$$\mathbb{D} = 9 - 4 \cdot 1 \cdot 6$$

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$$8) \quad y = x^2 - x$$

$$\underline{y = 3x - 3}$$

$$3x - 3 = x^2 - x$$

$$x^2 - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$

$$x_1 = 3 \quad y_1 = 6$$

$$x_2 = 1 \quad y_2 = 0$$

$$[x, y] = \{[3, 6], [1, 0]\}$$

$$2x - 8 = 3x^2 + x + 1$$

$$3x^2 - x + 9 = 0$$

$$D = 1 - 4 \cdot 3 \cdot 9$$

\emptyset

$$10) \quad y = 5x^2 - 7$$

$$y = 4x - 10$$

$$\underline{5x^2 - 7 = 4x - 10}$$

$$5x^2 - 4x + 3 = 0$$

$$D = 16 - 4 \cdot 3 \cdot 5 = 16 - 60$$

\emptyset

$$11) \quad y = 2 - 5x - 5x^2$$

$$y = 3x^2 + 7x + 2$$

$$\underline{2 - 5x - 5x^2 = 3x^2 + 7x + 2}$$

$$8x^2 + 12x = 0$$

$$2x^2 + 3x = 0$$

$$x(2x + 3) = 0$$

$$x_1 = 0 \quad x_2 = -\frac{3}{2}$$

$$y_1 = 2$$

$$y_2 = 3 \cdot \frac{9}{4} - \frac{21}{2} + 2 =$$

$$= \frac{27}{4} - \frac{42}{4} + \frac{8}{4} = -\frac{7}{4}$$

$$[x_1, y_1] = \left\{ [0, 2], \left[-\frac{3}{2}, -\frac{7}{4} \right] \right\}$$

$$12) \quad x + y = 7 \quad x = 7 - y$$

$$x^2 + y^2 = 37$$

$$\underline{49 - 14y + y^2 + y^2 = 37}$$

$$2y^2 - 14y + 12 = 0$$

$$y^2 - 7y + 6 = 0$$

$$(y-1)(y-6) = 0$$

$$y_1 = 1 \quad y_2 = 6$$

$$x_1 = 6 \quad x_2 = 1$$

$$[x_1, y_1] = \left\{ [6, 1], [1, 6] \right\}$$

$$13) xy + 21 = 0$$

$$\begin{array}{r} x - y = 10 \\ \hline x = 10 + y \end{array}$$

$$10y + y^2 + 21 = 0$$

$$y^2 + 10y + 21 = 0$$

$$(y+3)(y+7) = 0$$

$$y_1 = -3 \quad y_2 = -7$$

$$x_1 = 10 - 3 = 7 \quad x_2 = 10 - 7 = 3$$

$$[7, -3]; [3, -7]$$

$$14) x^2 + yx = 35$$

$$\begin{array}{r} x + 3y = 1 \\ \hline x = 1 - 3y \end{array}$$

$$(1 - 3y)^2 + y - 3y^2 = 35$$

$$1 - 6y + 9y^2 + y - 3y^2 - 35 = 0$$

$$6y^2 - 5y - 34 = 0$$

$$D = 25 - 4 \cdot 6 \cdot (-34) = 841$$

$$\sqrt{D} = 29 \quad y_{1,2} = \frac{5 \pm 29}{12} = \begin{cases} -2 \\ \frac{34}{12} = \frac{17}{6} \end{cases}$$

$$x_1 = 7$$

$$x_2 = -\frac{15}{2}$$

$$15) x^2 + y^2 - 4x - 6y - 3 = 0$$

$$\begin{array}{r} 2x - 3y - 3 = 0 \\ \hline x = \frac{3y + 3}{2} \end{array}$$

$$\frac{(3y+3)^2}{4} + y^2 - 2(3y+3) - 6y - 3 = 0$$

$$\frac{9y^2 + 18y + 9}{4} + y^2 - 6y - 6 - 6y - 3 = 0 / \cdot 4$$

$$9y^2 + 18y + 9 + 4y^2 - 48y - 36 = 0$$

$$13y^2 - 30y - 27 = 0$$

$$D = 900 - 4 \cdot 13 \cdot (-27) = 2304$$

$$\sqrt{D} = 48$$

$$y_{1,2} = \frac{30 \pm 48}{26} = \begin{cases} 3 \\ -\frac{18}{26} = -\frac{9}{13} \end{cases}$$

$$x_1 = 6$$

$$x_2 = \frac{3(-\frac{9}{13}) + 3}{2} = \frac{60}{13}$$

$$[6, 3]; [\frac{6}{13}, -\frac{9}{13}]$$

$$16) x - y - 5 = 0 \quad x = y + 5$$

$$x^2 + y^2 = 25$$

$$(y+5)^2 + y^2 = 25$$

$$y^2 + 10y + 25 + y^2 = 25$$

$$2y^2 + 10y = 0$$

$$y^2 + 5y = 0$$

$$y(y+5) = 0$$

$$y_1 = 0 \quad x_1 = 5$$

$$y_2 = -5 \quad x_2 = 0$$

$$[x_1 y_1] = \{ [5, 0], [0, -5] \}$$

$$17) \begin{array}{l} x^2 - y^2 = 5 \\ x + y - 5 = 0 \end{array} \rightarrow x = 5 - y$$

$$\frac{25 - 10y + y^2 - y^2 = 5}{20 = 10y}$$

$$y = 2 \quad [x, y] = [3, 2]$$

$$x = 3$$

$$18) \begin{array}{l} x^2 + y^2 = 5 \\ x - y = 3 \end{array} \quad x = 3 + y$$

$$\frac{9 + 6y + y^2 + y^2 = 5}{2y^2 + 6y + 4 = 0}$$

$$y^2 + 3y + 2 = 0$$

$$(y+1)(y+2) = 0$$

$$y_1 = -1 \quad y_2 = -2$$

$$x_1 = 2 \quad x_2 = 1$$

$$[x, y] = \{[-1, -1], [1, -2]\}$$

$$19) \begin{array}{l} y = x^2 \\ xy = 8 \end{array}$$

$$\frac{x \cdot x^2 = 8}{x^3 = 8}$$

$$x^3 = 2^3$$

$$x = 2 \quad y = 4 \quad [x, y] = [2, 4]$$

$$20) \begin{array}{l} x^2 + 9y^2 = 9 \\ x + 3y - 3 = 0 \end{array} \quad x = 3 - 3y$$

$$\frac{9 - 18y + 9y^2 + 9y^2 = 9}{18y^2 - 18y = 0}$$

$$y^2 - y = 0$$

$$y(y - 1) = 0$$

$$y_1 = 0 \quad y_2 = 1 \quad x_1 = 3 \quad x_2 = 0$$

$$[x, y] = [3, 0] \cup [0, 1]$$

$$21) \begin{array}{l} x^2 + y^2 = 13 \\ xy = 6 \end{array} \quad x = \frac{6}{y}$$

$$\frac{36}{y^2} + y^2 = 13 \quad \text{Substituien: } y^2 = a$$

$$\frac{36}{a} + a = 13 \quad | \cdot a$$

$$36 + a^2 = 13a$$

$$a^2 - 13a + 36 = 0$$

$$(a-4)(a-9) = 0$$

$$a_1 = 4$$

$$a_2 = 9$$

$$y_1 = 2$$

$$y_2 = -2$$

$$y_3 = 3$$

$$y_4 = -3$$

$$x_1 = 3$$

$$x_2 = -3$$

$$x_3 = 2$$

$$x_4 = -2$$

$$[3, 2]$$

$$[-3, -2]$$

$$[2, 3]$$

$$[-2, -3]$$

4x4

$$22) \quad 2x^2 - 3y^2 = 24$$

$$\begin{array}{l} 2x - 3y = 0 \\ \hline x = \frac{3y}{2} \end{array}$$

$$2 \cdot \frac{9y^2}{4} - 3y^2 = 24 \quad | \cdot 2$$

$$9y^2 - 6y^2 = 48$$

$$3y^2 = 48$$

$$y^2 = 16$$

$$y_1 = 4 \quad y_2 = -4$$

$$x_1 = 6$$

$$x_2 = -6$$

$$\{ [6, 4] ; [-6, -4] \} = [x, y]$$

$$23) \quad x^2 + y^2 = 4$$

$$\begin{array}{l} x - y + 4 = 0 \\ \hline x = y - 4 \end{array}$$

$$y^2 - 8y + 16 + y^2 = 4$$

$$2y^2 - 8y + 12 = 0$$

$$y^2 - 4y + 6 = 0$$

$$(y-2)^2 +$$

$$D = 4^2 - 4 \cdot 6 = 16 - 24 \quad \text{NR}$$

$$24) \quad 4x^2 - 9y^2 = 15$$

$$\begin{array}{l} 2x + 3y = 5 \\ \hline 2x = -3y + 5 \end{array}$$

$$(2x)^2 - 9y^2 = 15$$

$$(5 - 3y)^2 - 9y^2 = 15$$

$$25 - 30y + 9y^2 - 9y^2 = 15$$

$$-30y = -10$$

$$y = \frac{1}{3}$$

$$2x = -3 \cdot \frac{1}{3} + 5$$

$$2x = 4$$

$$x = 2$$

$$\underline{[2, \frac{1}{3}]}$$