Řešení:

Příklad	Diskriminant D	Kořeny x_1, x_2
a) $x^2 + 2x - 15 = 0$	$D = (-2)^2 + 4.1.(-15) = 64$	$x_1 = -5$; $x_2 = 3$
b) $x^2 - 11x + 28 = 0$	$D = (-11)^2 - 4.1.28 = 9$	$x_1 = 4$; $x_2 = 7$
c) $x^2 + 10x + 16 = 0$	$D = 10^2 - 4.1.16 = 36$	$x_1 = -8$; $x_2 = -2$
d) $x^2 - 25x - 84 = 0$	$D = (-25)^2 - 4.1.(-84) = 961$	$x_1 = -3$; $x_2 = 28$
e) $x^2 - 8x - 9 = 0$	$D = 64 + 36 = 100$	$x_1 = -1$; $x_2 = 9$
f) $x^2 + 25 = 0$	$D = 0 - 100 = -100$	Není řešení
g) $x^2 + 3x - 18 = 0$	$D = 9 + 72 = 81$	$x_1 = -6$; $x_2 = 3$
h) $x^2 - 14x - 15 = 0$	$D = 196 + 60 = 256$	$x_1 = -1$; $x_2 = 15$
i) $x^2 + 2x + 1 = 0$	$D = 4 - 4 = 0$	$x_1 = x_2 = -1$
j) $x^2 - 8x = 0$	$D = 64 - 0 = 64$	$x_1 = 0$; $x_2 = 8$
k) $x^2 - 81 = 0$	$D = 0 + 324 = 324$	$x_1 = -9$; $x_2 = 9$
l) $x^2 + 9x + 25 = 0$	$D = 81 - 100 = -19$	Není řešení

Řešte v R kvadratické rovnice:

7. $4x^2 - 8x = 0$ $/ : 4$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x_1 = 0 \quad x_2 = 2$

8. $5x^2 + x = 0$

$x(5x+1) = 0$

$x_1 = 0$

$x_2 = -\frac{1}{5}$

9. $4x^2 = 1$

$4x^2 - 1 = 0$

$(2x+1)(2x-1) = 0$

$x_1 = \frac{1}{2} \quad x_2 = -\frac{1}{2}$

10. $x^2 + 36 = 0$

NŘ

11. $2x^2 - 7x + 3 = 0$

$D = 49 - 4 \cdot 2 \cdot 3 = 49 - 24 = 25 \qquad \sqrt{D} = 5$

$x_{1,2} = \frac{7 \pm 5}{4} = \begin{cases} 3 \\ \frac{1}{2} \end{cases}$

12. $4x^2 - 20x + 25 = 0$

$D = 400 - 4 \cdot 4 \cdot 25 = 0$

$x_{1,2} = \frac{20 \pm 0}{8} = \frac{5}{2}$

13. $2x^2 + 10x + 17 = 0$

$D = 100 - 4 \cdot 2 \cdot 17 = 100 - 136 = -36$

NŘ

14. $4x^2 - 16x - 20 = 0 \quad / : 4$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x_1 = 5 \quad x_2 = -1$

15. $\frac{3}{t+3} + \frac{3-t}{t} = \frac{11}{10} \quad / \cdot 10t(t+3)$

$30t + 10(9 - t^2) = 11 \cdot t(t+3)$

$30t + 90 - 10t^2 = 11t^2 + 33t$

$21t^2 + 3t - 90 = 0$

$7t^2 + t - 30 = 0$

$D = 1 - 4 \cdot 7 \cdot (-30) = 841 \quad \sqrt{D} = 29$

$t_{1,2} = \frac{-1 \pm 29}{14} = \begin{cases} 2 \\ -\frac{15}{7} \end{cases}$

16. $\frac{1}{x}+\frac{1}{50-x}=\frac{1}{12}$ $/ 12x(50-x)$

$x \neq 0$
$x \neq 50$

$12(50-x)+12x=(50-x)x$

$600-12x+12x=50x-x^2$

$x^2-50x+600=0$

$D=(-50)^2-4\cdot 1\cdot 600=100$

$\sqrt{D}=10$

$x_{1,2}=\frac{50\pm 10}{2}=\begin{cases}30\\20\end{cases}$

17. $\frac{x^2-3x+2}{x^2+1}=0$ $/\cdot x^2+1$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$x_1=2$

$x_2=1$

18. $\frac{x+1}{x-3}+\frac{x+3}{x-1}=0$ $/\cdot (x-3)(x-1)$

$x \neq 3$
$x \neq 1$

$(x+1)(x-1)+(x+3)(x-3)=0$

$x^2-1+x^2-9=0$

$2x^2-10=0$

$x^2-5=0$

$(x+\sqrt{5})(x-\sqrt{5})=0$

$x_1=\sqrt{5}$
$x_2=-\sqrt{5}$

19. $\frac{1}{x}+\frac{4}{x+3}+\frac{4}{x-3}=0$ $/\cdot x(x+3)(x-3)$

$x \neq 0$
$x \neq \pm 3$

$x^2-9+4x(x-3)+4x(x+3)=0$

$x^2-9+4x^2-12x+4x^2+12x=0$

$9x^2-9=0$

$x^2-1=0$

$(x+1)(x-1)=0$

$x_1=-1$
$x_2=1$

20. $\frac{u+4}{u-4}-\frac{u+5}{u-5}-1=0$ $/(u-4)(u-5)$

$(u+4)(u-5)-(u+5)(u-4)-(u-4)(u-5)=0$

$u^2-u-20-u^2-u+20-u^2+9u-20=0$

$-u^2+7u-20=0$

$u^2-7u+20=0$

$D=49-4\cdot 1\cdot 20=-31$

NŘ v R

21. $\frac{1}{x+5}+0{,}2=\frac{-1}{5+2x}$ $\quad \nearrow \frac{2}{10}$ $\quad / \cdot 10(x+5)\cdot(5+2x)$

$10(5+2x)+(2x+10)(5+2x)=-10(x+5)$

$50+20x+10x+4x^2+50+20x=-10x-50$

$4x^2+60x+150=0$

$2x^2+30x+75=0$

$D=900-4\cdot 2\cdot 75=300$

$\sqrt{D}=10\sqrt{3}$

$x_{1,2}=\frac{-30\pm 10\sqrt{3}}{4}=\frac{\cancel{2}(-15\pm 5\sqrt{3})}{\cancel{4}\,2}$

$x_1=\frac{-15+5\sqrt{3}}{2}$ $\quad x_2=\frac{-15-5\sqrt{3}}{2}$

22. $\frac{4x+5}{x}-\frac{12}{x-2}=1$ $\quad / \cdot x(x-2)$

$(4x+5)(x-2)-12x=x^2-2x$

$4x^2-3x-10-12x=x^2-2x$

$3x^2-13x-10=0$

$D=(-13)^2-4\cdot 3(-10)=289$

$\sqrt{D}=17$

$x_{1,2}=\frac{13\pm 17}{6}=\begin{cases}5\\ -\frac{1}{3}\end{cases}$

$x\neq 0$

$x\neq 2$

23. $1+\frac{8}{x-4}-\frac{16}{x^2-16}=0$ $\quad / \cdot (x+4)(x-4)$ $\quad x\neq\pm 4$

$(x+4)(x-4)$

$x^2-16+8(x+4)-16=0$

$x^2-16+8x+32-16=0$

$x^2+8x=0$

$x(x+8)=0$ $\quad x_1=0$, $x_2=-8$

24. $\frac{x}{x+2}+\frac{1}{x-2}=\frac{11}{8}$ $\quad / (x+2)(x-2)\cdot 8$ $\quad x\neq\pm 2$

$8x(x-2)+8(x+2)=11(x^2-4)$

$8x^2-16x+8x+16=11x^2-44$

$3x^2+8x-60=0$

$D=64-4\cdot 3\cdot(-60)=784$

$\sqrt{D}=28$

$x_{1,2}=\frac{-8\pm 28}{6}=\begin{cases}\frac{10}{3}\\ -6\end{cases}$

25. $\frac{2x+1}{x+3}-\frac{x-1}{x^2-9}=\frac{x+3}{3-x}-\frac{4+x}{3+x}$ $\quad x\neq\pm 3$

$\frac{2x+1+4+x}{x+3}-\frac{x-1}{x^2-9}=-\frac{x+3}{x-3}$ $\quad / (x-3)(x+3)$

$(3x+5)(x-3)-x+1=-(x+3)^2$

$3x^2-4x-15-x+1=-(x^2+6x+9)$

$4x^2+x-5=0$

$D=1-4\cdot 4\cdot(-5)=81$

$\sqrt{D}=9$

$x_{1,2}=\frac{-1\pm 9}{8}=\begin{cases}-\frac{5}{4}\\ 1\end{cases}$

26. $\frac{2x-3}{x^3+1} = \frac{1}{x^2-x+1} - \frac{2}{x^2+2x+1}$ $/(x+1)^2(x^2-x+1)$

$x \neq -1$

$(2x-3)(x+1) = (x+1)^2 - 2x^2+2x-2$

$2x^2-x-3 = x^2+2x+1-2x^2+2x-2$

$3x^2-5x-2=0$

$D = 25-3\cdot 4(-2) = 49$

$\sqrt{D} = 7$

$x_{1,2} = \frac{5\pm 7}{6} = \begin{cases} 2 \\ -\frac{1}{3} \end{cases}$

Řešení kvadratických rovnic – komplikovanější úlohy - řešte v R :

27. $\frac{2x+2}{3x+7} = \frac{3x-2}{2x-7}$ $/(2x-7)(3x+7)$

$x \neq \frac{7}{2}$ $x \neq -\frac{7}{3}$

$4x^2-10x-14 = 9x^2+15x-14$

$5x^2+25x = 0$

$x^2+5x = 0$

$x(x+5)=0$

$x_1 = 0$ $x_2 = -5$

28. $5x^2 + 10x - 36 = -3(x+2)^2 + 24x - 23$

$5x^2+10x-36 = -3x^2-12x-12+24x-23$

$8x^2-2x-1=0$

$D = (-2)^2-4\cdot 8\cdot(-1) = 36$

$\sqrt{D} = 6$

$x_{1,2} = \frac{2\pm 6}{16} = \begin{cases} \frac{1}{2} \\ -\frac{1}{4} \end{cases}$

29. $\frac{x}{x+1} + \frac{3x}{x-1} = \frac{5x^2-8}{x^2-1}$ $/(x^2-1)$

$x \neq \pm 1$

$x(x-1)+3x(x+1) = 5x^2-8$

$x^2-x+3x^2+3x = 5x^2-8$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x_1 = 4$ $x_2 = -2$

30. $\frac{3}{3+x} + \frac{3-x}{x} = \frac{11}{10}$ $/\cdot 10x(3+x)$

$x \neq 0$

$x \neq -3$

$30x+10(9-x^2) = 11x(3+x)$

$30x+90-10x^2 = 33x+11x^2$

$21x^2+3x-90=0$

$7x^2+x-30=0$

$D = 1-4\cdot 7(-30) = 841$

$\sqrt{D} = 29$

$x_{1,2} = \frac{-1\pm 29}{14} = \begin{cases} -\frac{15}{7} \\ 2 \end{cases}$

31. $\frac{x-1}{x-2} + \frac{x-2}{x-1} = \frac{5}{2}$ $/\cdot 2(x-1)(x-2)$

$x \neq 1$

$x \neq 2$

$2(x-1)^2+2(x-2)^2 = 5(x-2)(x-1)$

$2x^2-4x+2+2x^2-8x+8 = 5x^2-15x+10$

$x^2-3x = 0$

$x(x-3)=0$

$x_1 = 0$ $x_2 = 3$

32. $\frac{x+3}{x-3} + \frac{x-6}{x+6} = \frac{11}{5}$ $/\cdot 5(x-3)(x+6)$

$5(x+3)(x+6)+5(x-3)(x-6) = 11(x-3)(x+6)$

$5x^2+45x+90+5x^2-45x+90 = 11x^2+33x-198$

$10x^2+180 = 11x^2+33x-198$

$x^2+33x-378=0$

$D = 33^2+4\cdot 378 = 2601$ $\sqrt{D} = 51$

$x_{1,2} = \frac{-33\pm 51}{2} = \begin{cases} -42 \\ 9 \end{cases}$

Rozklad kvadratického trojčlenu – krácení zlomků

Kraťte zlomky a udejte podmínky, za kterých mají smysl:

33. $\frac{4x^2+11x-3}{2x^2+7x+3} = \left(\frac{4x-1}{2x+1}\right)$ $= \frac{(x+3)(4x-1)}{(x+3)(2x+1)}$

Č: $D = 121 + 4\cdot4\cdot3 = 169$ $\sqrt{D} = 13$ $x_{1,2} = \frac{-11 \pm 13}{8} = \begin{cases} -3 \\ \frac{1}{4} \end{cases}$

J: $D = 49 - 4\cdot2\cdot3 = 25$ $\sqrt{D} = 5$ $x_{1,2} = \frac{-7 \pm 5}{4} = \begin{cases} -3 \\ -\frac{1}{2} \end{cases}$

34. $\frac{8x^2+10x+3}{12x^2+5x-3} = \left(\frac{2x+1}{3x-1}\right)$ $\frac{(4x+3)(2x+1)}{(4x+3)(3x-1)}$

Č: $D = 100 - 4\cdot8\cdot3 = 100 - 96 = 4$ $\sqrt{D} = 2$ $x_{1,2} = \frac{-10 \pm 2}{16} = \begin{cases} -\frac{3}{4} \\ -\frac{1}{2} \end{cases}$

J: $D = 25 - 4(12)(-3) = 25 + 144 = 169$ $\sqrt{D} = 13$ $x_{1,2} = \frac{-5 \pm 13}{24} = \begin{cases} -\frac{3}{4} \\ \frac{1}{3} \end{cases}$

35. $\frac{6x^2+35x-6}{3x^2+20x+12} = \left(\frac{6x-1}{3x+2}\right)$ $\frac{(x+6)(6x-1)}{(x+6)(3x+2)}$

Č: $D = 35^2 + 4\cdot36 = 1369$ $\sqrt{D} = 37$ $x_{1,2} = \frac{-35 \pm 37}{12} = \begin{cases} \frac{1}{6} \\ -6 \end{cases}$

J: $D = 400 - 12\cdot12 = 256$ $\sqrt{D} = 16$ $x_{1,2} = \frac{-20 \pm 16}{6} = \begin{cases} -6 \\ -\frac{2}{3} \end{cases}$

36. $\frac{2x^2+15x+7}{3x^2+22x+7} = \left(\frac{2x+1}{3x+1}\right)$ $\frac{(2x+1)(x+7)}{(3x+1)(x+7)}$

Č: $D = 225 - 4\cdot2\cdot7 = 169$ $\sqrt{D} = 13$ $x_{1,2} = \frac{-15 \pm 13}{4} = \begin{cases} -7 \\ -\frac{1}{2} \end{cases}$

J: $D = 22^2 - 12\cdot7 = 400$ $\sqrt{D} = 20$ $x_{1,2} = \frac{-22 \pm 20}{6} = \begin{cases} -7 \\ -\frac{1}{3} \end{cases}$

37. $\frac{2x^2-3x-2}{3x^2-2x-8} = \left(\frac{2x+1}{3x+4}\right)$ $\frac{(x-2)(2x+1)}{(x-2)(3x+4)}$

Č: $D = 9 - 4\cdot2\cdot(-2) = 25$ $\sqrt{D} = 5$ $x_{1,2} = \frac{3 \pm 5}{4} = \begin{cases} 2 \\ -\frac{1}{2} \end{cases}$

J: $D = 4 - 4\cdot3(-8) = 100$ $\sqrt{D} = 10$ $x_{1,2} = \frac{2 \pm 10}{6} = \begin{cases} 2 \\ -\frac{4}{3} \end{cases}$

38. $\frac{4x^2-7x-2}{3x^2-2x-8} = \left(\frac{4x+1}{3x+4}\right)$ $\frac{(x-2)(4x+1)}{(x-2)(3x+4)}$

Č: $D = 49 - 4\cdot4(-2) = 81$ $\sqrt{D} = 9$ $x_{1,2} = \frac{7 \pm 9}{8} = \begin{cases} 2 \\ -\frac{1}{4} \end{cases}$

J: $D = 4 - 4\cdot3(-8) = 100$ $\sqrt{D} = 10$ $x_{1,2} = \frac{2 \pm 10}{6} = \begin{cases} 2 \\ -\frac{4}{3} \end{cases}$