Částečné odmocňování a usměrňování zlomků

1. Částečně odmocněte:

1.1. $\sqrt{8},\ \sqrt{12},\ \sqrt{20},\ \sqrt{72},\ \sqrt{125}$

1.2. $\sqrt[3]{32},\ \sqrt[3]{54},\ \sqrt[3]{500},\ \sqrt[3]{3\,000},\ \sqrt[3]{2\,160}$

1.3. $\sqrt[3]{a^4},\ \sqrt[4]{b^{11}},\ \sqrt[5]{c^{39}},\ \sqrt{d^{25}},\ \sqrt{40\ e^9\, f^{11}\ g^{23}\ h^4}$

2. Vypočtěte:

2.1. $\sqrt{8} + \sqrt[3]{8} + \sqrt{12} + \sqrt[4]{16}$

2.2. $7\sqrt{44} + 6\sqrt{99}$

2.3. $15\sqrt[3]{750} - 11\sqrt[3]{48}$

2.4. $3\sqrt{2} - 4\sqrt{50} + \sqrt{18}$

2.5. $\sqrt{a^3} - 2a\sqrt{a} + 3\sqrt{a^3}$

3. Usměrněte zlomky:

3.1. $\frac{1}{\sqrt{3}},\ \frac{2}{\sqrt{2}},\ \frac{5}{4\sqrt{5}},\ -\frac{55}{\sqrt{11}},\ \frac{18}{5\sqrt{3}}$

3.2. $\frac{7k}{\sqrt{k}},\ \frac{1}{\sqrt[11]{l^8}},\ \frac{m}{\sqrt[3]{m}},\ \frac{n^2}{\sqrt[5]{n^3}},\ \frac{10p}{\sqrt[5]{p^7}}$

3.3. $\left(\sqrt{5} - 2\right)^{-1},\ \frac{1-\sqrt{3}}{1+\sqrt{3}},\ \frac{2}{3\sqrt{5}-\sqrt{13}},\ \frac{7\sqrt{3}}{\sqrt{5}+2\sqrt{3}},\ \frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{2}+\sqrt{3}}$

ŘEŠENÍ

1. Částečné odmocňování

1.1. Částečně odmocněte:

$\sqrt{8} = \sqrt{4{\cdot}2} = 2\sqrt{2}$

$\sqrt{12} = \sqrt{4{\cdot}3} = 2\sqrt{3}$

$\sqrt{20} = \sqrt{4{\cdot}5} = 2\sqrt{5}$

$\sqrt{72} = \sqrt{36{\cdot}2} = 6\sqrt{2}$

$\sqrt{125} = \sqrt{25{\cdot}5} = 5\sqrt{5}$

1.2. Částečně odmocněte:

$\sqrt[3]{32} = \sqrt[3]{8{\cdot}4} = 2\sqrt[3]{4}$

$\sqrt[3]{54} = \sqrt[3]{27{\cdot}2} = 3\sqrt[3]{2}$

$\sqrt[3]{500} = \sqrt[3]{125{\cdot}4} = 5\sqrt[3]{4}$

$\sqrt[3]{3\,000} = \sqrt[3]{1\,000{\cdot}3} = 10\sqrt[3]{3}$

$\sqrt[3]{2\,160} = \sqrt[3]{216{\cdot}10} = 6\sqrt[3]{10}$

1.3. Částečně odmocněte:

$\sqrt[3]{a^4} = \sqrt[3]{a^3{\cdot}a} = a\sqrt[3]{a} \qquad a \geq 0$

$\sqrt[4]{b^{11}} = \sqrt[4]{b^8{\cdot}b^3} = b^2\sqrt[4]{b^3} \qquad b \geq 0$

$\sqrt[5]{c^{39}} = \sqrt[5]{c^{35}{\cdot}c^4} = c^7\sqrt[5]{c^4} \qquad c \geq 0$

$\sqrt{d^{25}} = \sqrt{d^{24}{\cdot}d} = d^{12}\sqrt{d} \qquad d \geq 0$

$\sqrt{40\,e^9\,f^{11}\,g^{23}\,h^4} = \sqrt{4{\cdot}10{\cdot}e^8e^1f^{10}f^1g^{22}g^1h^4} = 2e^4f^5g^{11}h^2\sqrt{10efg}$

$e \geq 0, f \geq 0, g \geq 0, h \in R$

2. Vypočtěte:

2.1. $\sqrt{8}+\sqrt[3]{8}+\sqrt{12}+\sqrt[4]{16}=\sqrt{4\cdot 2}+2+\sqrt{4\cdot 3}+2=4+2\sqrt{2}+2\sqrt{3}$

2.2. $7\sqrt{44}+6\sqrt{99}=7\sqrt{4\cdot 11}+6\sqrt{9\cdot 11}=14\sqrt{11}+18\sqrt{11}=32\sqrt{11}$

2.3. $15\sqrt[3]{750}-11\sqrt[3]{48}=15\sqrt[3]{125\cdot 6}-11\sqrt[3]{8\cdot 6}=75\sqrt[3]{6}-22\sqrt[3]{6}=53\sqrt[3]{6}$

2.4. $3\sqrt{2}-4\sqrt{50}+\sqrt{18}=3\sqrt{2}-4\sqrt{25\cdot 2}+\sqrt{9\cdot 2}=3\sqrt{2}-20\sqrt{2}+3\sqrt{2}=-14\sqrt{2}$

2.5. $\sqrt{a^3}-2a\sqrt{a}+3\sqrt{a^3}=\sqrt{a^2a^1}-2a\sqrt{a}+3\sqrt{a^2a^1}=a\sqrt{a}-2a\sqrt{a}+3a\sqrt{a}=2a\sqrt{a}$

$a\geq 0$

3. Usměrňování zlomků:

3.1. Usměrněte zlomky:

$$\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$$\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{2}}{2}=\sqrt{2}$$

$$\frac{5}{4\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{5\sqrt{5}}{20}=\frac{\sqrt{5}}{4}$$

$$-\frac{55}{\sqrt{11}}\cdot\frac{\sqrt{11}}{\sqrt{11}}=-\frac{55\sqrt{11}}{11}=-5\sqrt{11}$$

$$\frac{18}{5\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{18\sqrt{3}}{15}=\frac{6\sqrt{3}}{5}$$

3.2. Usměrněte zlomky:

$$\frac{7k}{\sqrt{k}}\cdot\frac{\sqrt{k}}{\sqrt{k}}=\frac{7k\sqrt{k}}{k}=7\sqrt{k}\qquad k>0$$

$$\frac{1}{\sqrt[11]{l^8}}\cdot\frac{\sqrt[11]{l^3}}{\sqrt[11]{l^3}}=\frac{\sqrt[11]{l^3}}{l}\qquad l>0$$

$$\frac{m}{\sqrt[3]{m}}\cdot\frac{\sqrt[3]{m^2}}{\sqrt[3]{m^2}}=\frac{m\sqrt[3]{m^2}}{m}=\sqrt[3]{m^2}\qquad m>0$$

$$\frac{n^2}{\sqrt[5]{n^3}} \cdot \frac{\sqrt[5]{n^2}}{\sqrt[5]{n^2}} = \frac{n^2\sqrt[5]{n^2}}{n} = n\sqrt[5]{n^2} \qquad n > 0$$

$$\frac{10p}{\sqrt[5]{p^7}} \cdot \frac{\sqrt[5]{p^3}}{\sqrt[5]{p^3}} = \frac{10p\sqrt[5]{p^3}}{p^2} = \frac{10\sqrt[5]{p^3}}{p} \qquad p > 0$$

3.3. Usměrněte zlomky:

$$\left(\sqrt{5}-2\right)^{-1} = \frac{1}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \frac{\sqrt{5}+2}{1} = \sqrt{5}+2$$

$$\frac{1-\sqrt{3}}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{\left(1-\sqrt{3}\right)^2}{1-3} = \frac{1-2\sqrt{3}+3}{-2} = \frac{4-2\sqrt{3}}{-2}$$

$$= \frac{-2\left(\sqrt{3}-2\right)}{-2} = \sqrt{3}-2$$

$$\frac{2}{3\sqrt{5}-\sqrt{13}} \cdot \frac{3\sqrt{5}+\sqrt{13}}{3\sqrt{5}+\sqrt{13}} = \frac{2\left(3\sqrt{5}+\sqrt{13}\right)}{45-13} = \frac{2\left(3\sqrt{5}+\sqrt{13}\right)}{32}$$

$$= \frac{3\sqrt{5}+\sqrt{13}}{16}$$

$$\frac{7\sqrt{3}}{\sqrt{5}+2\sqrt{3}} \cdot \frac{\sqrt{5}-2\sqrt{3}}{\sqrt{5}-2\sqrt{3}} = \frac{7\sqrt{3}\left(\sqrt{5}-2\sqrt{3}\right)}{5-12} = \frac{-7\left(6-\sqrt{15}\right)}{-7}$$

$$= 6-\sqrt{15}$$

$$\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{2}+\sqrt{3}} \cdot \frac{2\sqrt{2}-\sqrt{3}}{2\sqrt{2}-\sqrt{3}} = \frac{4\sqrt{6}-6-4+\sqrt{6}}{8-3} = \frac{5\sqrt{6}-10}{5}$$

$$= \frac{5\left(\sqrt{6}-2\right)}{5} = \sqrt{6}-2$$